Semi-Implicit Data-Driven Predictive Control for Agile Flying and Beyond

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***Abstract*—State-of-the-art model-based control strategies have demonstrated success in enabling dynamic locomotion behaviors such as flying, hopping, and walking in robotic systems. However, the performance of these behaviors in practice remains inadequate, particularly due to the inherent discrepancies between the modeled dynamics and the physical hardware, which inevitably lead to trajectory tracking errors. To mitigate this issue, we propose a semi-implicit control framework that bridges the *model-to-real gap* by incorporating a data-driven control approach combined with the existing model-based design. We validate the proposed method on PogoX, a custom-designed multi-modal locomotion robot, demonstrating high-precision hopping and flying behaviors in both simulation and real-world experiments. This semi-implicit control paradigm offers a generalizable solution for improving performance across a broad range of robotic platforms and locomotion behaviors.**

I. Introduction

Model-based control remains the foundation for achieving reliable, efficient, and safe behaviors in modern robotic systems, particularly in flying [1], [2] and legged [3], [4] platforms. These controllers rely on dynamic models—ranging from simplified abstractions to full-order Lagrangian formulations—that capture how input forces or torques influence the robot's states. Although grounded in physics, these models inevitably deviate from reality due to unmodeled complexities such as structural compliance, actuator limitations, and sensing or computational delays [5], resulting in a persistent *model-to-real gap*.

To mitigate this gap, implicit control approaches [6]–[9] that leverage data for control have emerged as alternatives, learning system dynamics [7] or control policies [8] directly from data. However, these methods often replace the original model-based controller, thereby forfeiting its theoretical guarantees [10].

In this work, we aim to utilize the data-driven predict control (DDPC) [11] techniques but keep the original model-based control design intact to address tracking errors caused by the model-to-real gap in robotic locomotion control. This pipeline is named as the *semi-implicit control* framework as illustrated in Fig. 1. We assume a model-based control design, presented by the dashed box, has been synthesized to realize robotic locomotion by performing trajectory tracking. The model-to-real gap causes errors between the reference and resultant trajectories. We then take a perspective of treating the reference trajectory as *the input* and the resultant trajectory as *the output* of the dashed-boxed system, for which we apply the

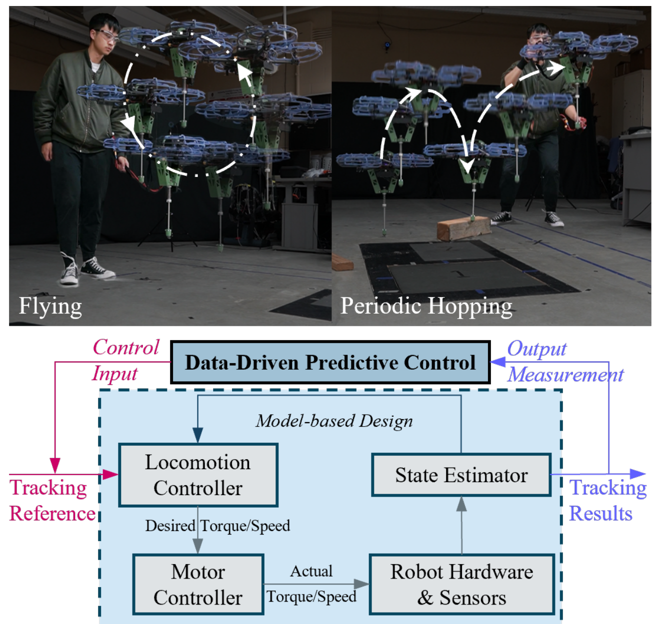

Fig. 1. Overview of the semi-implicit control approach (bottom) and the application on PogoX (top).

data-driven predictive control which is a data-driven control method to address the model discrepancy and realize hyper-accurate trajectory tracking.

We leverage semi-implicit control to online realize precise flying and hybrid locomotion behaviors like hopping of the robot PogoX [12] in both simulation and hardware. The original flying and hopping behaviors are realized by model-based controllers, which produce inaccurate tracking due to the model-to-real gap. We show that by utilizing the proposed framework, the model-to-real gap is bridged and the tracking accuracy is significantly improved. Moreover, to enable predictive control on hybrid locomotion behaviors, we present an artificial input-output (IO) data generation process so that a uniform DDPC controller can be used to realize control over hybrid dynamics.

II. Semi-Implicit DDPC of Flying

In this section and the next, we describe the semi-implicit control approach via DDPC for realizing flying of PogoX [12].

A. Linear Flying Dynamics and Controller

We show that the closed-loop IO dynamics of PogoX during flying can be approximated by an LTI system, which rationalizes the application of DDPC for semi-implicit control.

Flying Dynamics: By assuming: (a) the robot operates with small roll ϕ and pitch θ angles, where $\cos\phi \approx 1, \sin\phi \approx \phi, \cos\theta \approx 1, \sin\theta \approx \theta$; and (b) the cross product term of the angular velocity is relatively small, robot dynamics is approximately a linear dynamics [13]. The thrust force and moment generated by the rotor can be modeled as $\mathcal{F} = k_F \omega_{\text{rotor}}^2$, where k_F is constant determined by the propeller, and ω_{rotor} is the rotor velocity [14]. Therefore, the motor dynamics and its controller are all treated as linear.

Model-Based Flying Control Design: By assuming the robot does not turn for simplicity, i.e. $\psi = 0$, the orientation is represented with ZYX Euler angle. We choose the Euler angles and total thrust, $(\psi, \theta, \phi, F_{\text{thrust}}^{\sum})$, as the output of the flight controller. To realize flying, a cascaded linear PD controller is designed to stabilize the pose tracking [15].

B. Semi-Implicit Control via DDPC

We present a computationally efficient DDPC formulation for realizing online semi-implicit control.

To apply the DDPC, we first off-line generate sufficient IO data of the closed-loop system to realize flying behaviors to construct the data-transition matrix offline. Then, we use the data-transition matrix for semi-implicit control through DDPC:

$$\min_{u,y,\sigma_y} ||y - y^{\text{des}}||_Q^2 + ||u||_R^2 + \lambda_\sigma ||\sigma_y||^2, \tag{1}$$

$$\text{s.t.} \quad \begin{bmatrix} y \\ \sigma_y \end{bmatrix} = \mathcal{G} \begin{bmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \end{bmatrix}, u \in \mathcal{U}, y \in \mathcal{Y},$$

where y^{des} is the desired trajectory, u and y are commanded trajectory and realized trajectory. $\mathcal{G}$ is named as the data-transition matrix [16] and is computed offline. Solving this QP online yields the control input u to steer the realized output y towards the desired one y^{des}.

III. Semi-Implicit DDPC of Hybrid Locomotion

We propose a semi-implicit control strategy using DDPC to achieve periodic hopping locomotion on PogoX. The robot's hybrid dynamics—comprising distinct aerial and ground phases—pose a challenge to applying continuous predictive control throughout the motion. To address this, we introduce an "artificial input-output (IO)" trajectory generation scheme during the ground phase, enabling DDPC to perform continuous control in the aerial phase.

A. Hybrid Dynamics and Hopping Control

Hybrid Dynamics of Hopping: Hopping involves hybrid dynamics with two phases: aerial and ground [17]. The aerial phase follows quadrotor dynamics, while the ground phase includes ground reaction forces under a no-slip foot contact assumption. Transitions are governed by a discrete impact map (aerial to ground) and smooth lift-off (ground to aerial).

Model-Based Hopping Control Design: The hopping controller is split into vertical height and leg angle control. Desired vertical trajectories are optimized using the robot's vertical dynamics. Leg angles are modulated based on prior work [12] and are used to regulate horizontal velocity through step-to-step (S2S) dynamics-based control [18].

B. Semi-Implicit Control for Hybrid Locomotion

Due to the hybrid nature of hopping, the data-transition matrix and IO data used for flying tasks cannot be directly applied to hopping, as the underlying dynamics differ. However, since control is mainly applied during the aerial phase—where the dynamics match those of flight—we focus on constructing artificial IO data during the ground phase that mimics aerial dynamics. By combining this with real IO data from the aerial phase, we obtain continuous IO trajectories that reflect only the aerial dynamics.

Artificial IO Data Generation: Our goal is to design $u_{g-\text{ini}}, y_{g-\text{ini}}$ that artificially represent the ground trajectories using aerial dynamics. For each hopping, we formulate this QP problem:

$$\min_{u_{g-\text{ini}}, y_{g-\text{ini}}, g} ||g||^2, \tag{2}$$

$$\text{s.t.} \begin{bmatrix} U_p \\ Y_p \end{bmatrix} g = \begin{bmatrix} u_{<d,g,a>\text{ini}} \\ y_{<d,g,a>\text{ini}} \end{bmatrix}, u \in \mathcal{U}, y \in \mathcal{Y},$$

where U_p, Y_p are the Hankel matrices [11] representing the aerial dynamics, and the subscripts d, g, a denote the descending, ground, and ascending phases, respectively. After generating the initial ground-phase trajectories, an augmented Hankel matrix is constructed by combining real aerial-phase data with the generated ground-phase data.

Semi-Implicit Control Formulation for Hopping: The DDPC problem, now including the ground phase, follows the same formulation as in (1). The matrix $\mathcal{G}$ is computed offline using the augmented Hankel matrix, enabling the QP to be solved online in real time.

IV. Results

In this section, we will present the results of our approach for realizing trajectory tracking on PogoX in both simulation and hardware. The hardware control is realized in ROS environment.

A. Semi-Implicit Control for Flying

We first present the results of applying semi-implicit control to the flying of PogoX. The input data are collected from the desired z and θ trajectories that are mapped from RC control inputs. The output data come from the direct measurement of state estimator. The lower-level controller and its feedback gains during data collection and experiment remain the same.

The robot is commanded to follow an ellipsoid trajectory in its sagittal plane where the desired $x-$direction position is transformed to the desired leg angle via a PD controller. The initial trajectory length is set to 20, while the prediction horizon is kept relatively short to 15 to enhance computational efficiency.

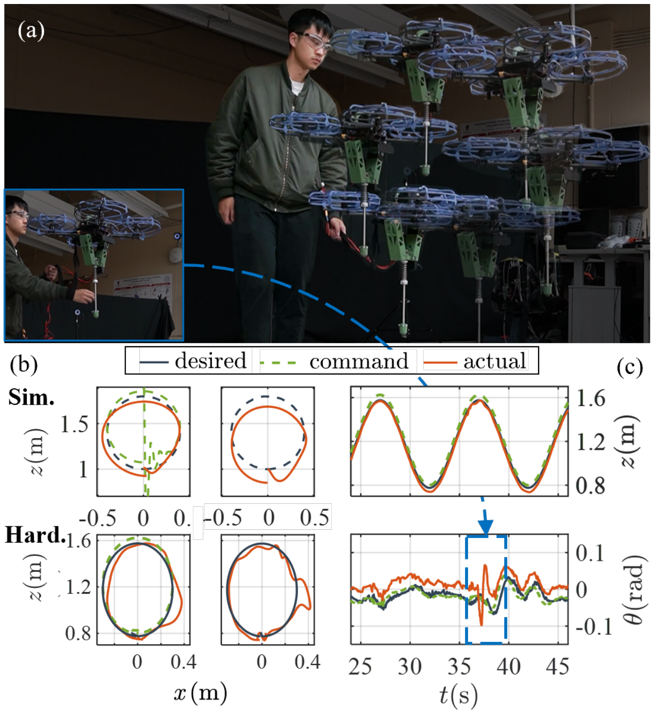

Fig. 2. Results on flying: (a) depicts PogoX in flying and disturbance injection; (b) compares the quality of trajectory tracking in $x - z$ plane; and (c) demonstrates how DDPC responds to disturbances in both height and leg angle tracking.

Simulation: The height and leg angle reference tracking performance can be seen in Fig. 2. As shown in the $x - z$ plot, the quadrotor failed to follow the correct trajectory due to the introduced modeling error, and semi-implicit control via DDPC steers it back to the desired height.
Hardware: The same controller and desired trajectory are used on the hardware with additional disturbance forces applied to the robot leg to disrupt both height and leg angle tracking. The comparison of the robustness of the controller, with and without DDPC, is shown in Fig. 2.

B. Semi-Implicit Control for Hopping

We now present the results of the periodic hopping behaviors. During both the data collection and experimental evaluation, the robot is commanded to perform periodic hopping, aiming to realize a desired apex height and a target horizontal velocity. To ensure adequate coverage of a complete hopping cycle, the prediction horizon is set to 25, and the initial trajectory length is maintained at 20.
Simulation: Fig. 3 illustrates the performance of hopping behavior in simulation, with and without DDPC. It is evident that with DDPC, height tracking is significantly improved, while the leg angles maintain sufficiently accurate tracking performance. With DDPC, the reference trajectories are steered to higher targets to account for the tracking inaccuracies during lift-off.
Hardware: The height and leg angle reference tracking performance can be seen in Fig. 3. Similar to the simulation results, the height and leg angle tracking have been significantly improved, leading to better overall stabilization of step-to-step

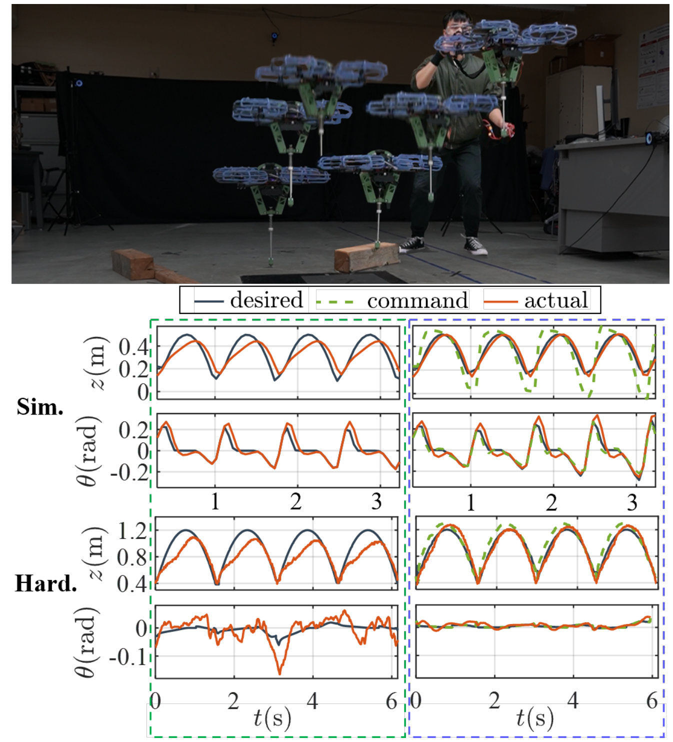

Fig. 3. Results on periodic hopping: trajectories without reference-steering (green) and with reference-steering (blue).

(S2S) dynamics and hopping behaviors. The steered output demonstrates that DDPC can predict the mismatch between the reference and actual trajectories, adjusting the reference trajectory to better track the desired trajectories.

V. Future Work

Future work will focus on integrating the implicitly modeled dynamics into the standard MPC framework to more accurately capture the effects of robot-environment interaction (REI), with particular emphasis on non-rigid contact dynamics. This integration aims to enhance the controller's ability to reason about complex and compliant terrain interactions.

Furthermore, the proposed semi-implicit control framework will be evaluated on more advanced robotic platforms and behaviors, including an aerial-terrestrial variant of PogoX equipped with a skate shoe consisting of two wheels beneath the pogo stick, as illustrated in Fig. 4. This evaluation will test the ability of the control pipeline to operate across multi-modal locomotion, requiring smooth transitions and unified control across both aerial, intermittent contact and continuous contact phases.

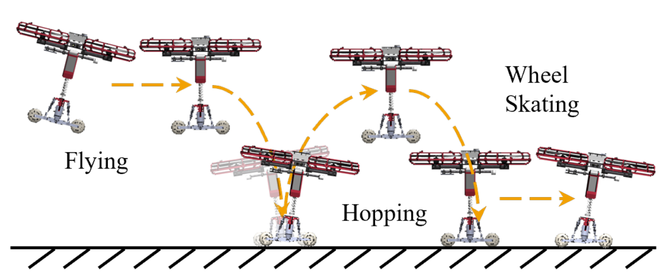

Fig. 4. Locomotion of aerial-terrestrial version of PogoX.

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